4.1 The Coordinate Plane

Goal Plot points in a coordinate plane.

VOCABULARY

Coordinate plane

Origin

x-axis

y-axis

Ordered pair

x-coordinate

y-coordinate

Quadrant

Scatter plot

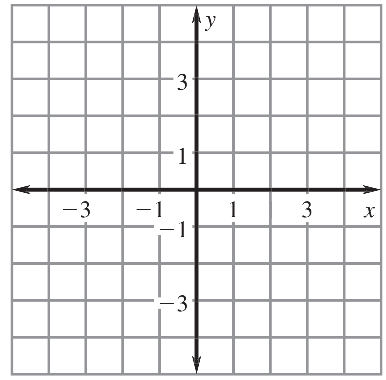
Example 1 *Plot Points in a Coordinate Plane*

Plot the points $A(-2, 3)$, $B(3, -4)$, and $C(0, -2)$ in a coordinate plane.

To plot the point $A(-2, 3)$, start at the _______. Move 2 units to the ______ and 3 units _______.

To plot the point $B(3, -4)$, start at the _______. Move 3 units to the ______ and 4 units _______.

To plot the point $C(0, -2)$, start at the _______. Move 0 units to the ______ and 2 units _______.



Example 2 *Identify Quadrants*

Name the quadrants the points $D(-2, -9)$ and $E(12, 4)$ are in.

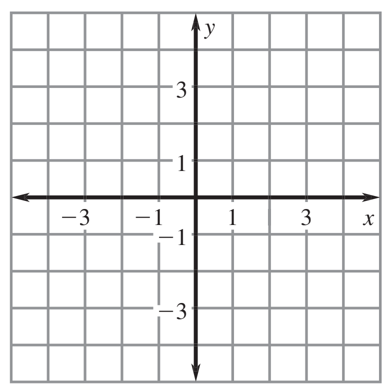
The point $D(-2, -9)$ is in Quadrant ___ because its x- and y-coordinates are both _________.

The point $E(12, 4)$ is in Quadrant ___ because its x- and y-coordinates are both _________.

Checkpoint **Plot the points in the same coordinate plane.**

1. $A(-3, -2)$ **2.** $B(4, 0)$

3. $C(1, 4)$ **4.** $D(-3, 2)$



Name the quadrant the point is in.

5. $(-6, 7)$	**6.** $(-6, -7)$	**7.** $(6, -7)$	**8.** $(6, 7)$

Example 3 *Make a Scatter Plot*

NCAA Basketball Teams The number of NCAA men's college basketball teams is shown in the table.

Year	1995	1996	1997	1998	1999	2000
Men's teams	868	866	865	895	926	932

a. Make a scatter plot of the data.

b. Describe the pattern of the number of men's basketball teams.

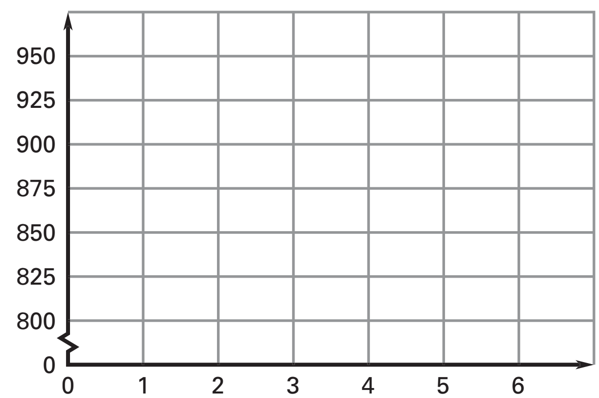
Solution

a. Let M represent ______________________________. Let t represent _________________________________.

Because you want to see how the number of teams changed over time, put t on the ___________ axis and M on the ___________ axis.

Choose a scale. Use a break in the scale for the number of teams to focus on the values between _____ and _____.

NCAA Men's Basketball Teams



b. From the scatter plot, you can see that the number of men's teams in the NCAA was ________________ for three years and then began to ____________________.

4.2 Graphing Linear Equations

Goal Graph a linear equation using a table of values.

VOCABULARY

Linear equation

Solution of an equation

Function form

Graph of an equation

Example 1 ***Check Solutions of Linear Equations***

Determine whether the ordered pair is a solution of $2x + 3y = -6$.

a. $(3, -4)$ **b.** $(-4, 1)$

Solution

a. $2x + 3y = -6$ **Write original equation.**

$2(\_\_\_) + 3(\_\_\_\_) \stackrel{?}{=} -6$ **Substitute ___ for *x* and ____ for *y*.**

$\_\_\_\_ \ \_\_\_ \ -6$ **Simplify. __________ statement.**

Answer $(3, -4)$ ______ a solution of the equation $2x + 3y = -6$.

b. $2x + 3y = -6$ **Write original equation.**

$2(\_\_\_\_) + 3(\_\_\_) \stackrel{?}{=} -6$ **Substitute ____ for *x* and ___ for *y*.**

$\_\_\_\_ \ \_\_\_ \ -6$ **Simplify. __________ statement.**

Answer $(-4, 1)$ ______ a solution of the equation $2x + 3y = -6$.

Checkpoint Determine whether the ordered pair is a solution of $-2x + y = 3$.

1. (0, 3)	2. (1, 1)	3. (1, 5)

Example 2 *Find Solutions of Linear Equations*

Find three ordered pairs that are solutions of $-5x + y = -2$.

1. **Rewrite** the equation in function form to make it easier to substitute values into the equation.

   $-5x + y = -2$ **Write original equation.**

   $y =$ ________ **Add ____ to each side.**

2. **Choose** any value for x and substitute it into the equation to find the corresponding y-value. The easiest x-value is ___.

   $y = 5(\_\_) - 2$ **Substitute ___ for x.**

   $y =$ ____ **Simplify. The solution is ________.**

3. **Select** a few more values of x and make a table to record the solutions.

x	0	1	2	3	−1	−2
y						

Answer ________, ______, and __________ are three solutions of $-5x + y = -2$.

GRAPHING A LINEAR EQUATION

Step 1 **Rewrite** the equation in ________ form, if necessary.

Step 2 **Choose** a few values of x and make a ____________.

Step 3 **Plot** the points from the table of values. A line through these points is the ______ of the equation.

Example 3 *Graph a Linear Equation*

Use a table of values to graph the equation $x + 4y = 4$.

1. **Rewrite** the equation in function form by solving for y.

$x + 4y = 4$	**Write original equation.**
$4y =$ ____ $+ 4$	**Subtract ___ from each side.**
$y = \frac{\qquad}{\qquad}$	**Divide each side by ___.**

2. **Choose** a few values of x and make a table of values.

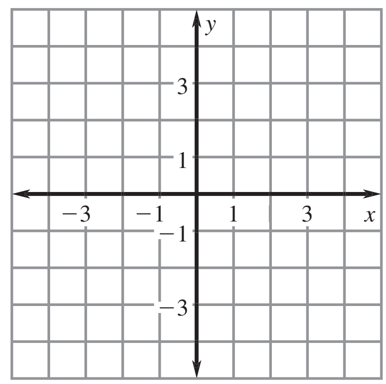
When graphing a linear equation, try choosing values of x that include negative values, zero, and positive values to see how the graph behaves to the left and right of the y-axis.

x	-4	0	4
y			

You have found three solutions.

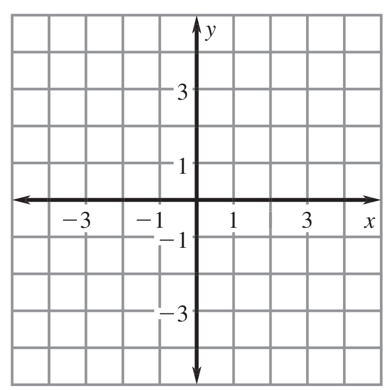
$(-4,$ __ $)$, $(0,$ __ $)$, $(4,$ __ $)$

3. **Plot** the points and draw a line through them.



✔ ***Checkpoint*** **Complete the following exercise.**

4. Use a table of values to graph the equation $x - 2y = 1$.



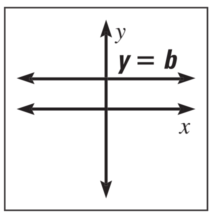
4.3 Graphing Horizontal and Vertical Lines

Goal Graph horizontal and vertical lines.

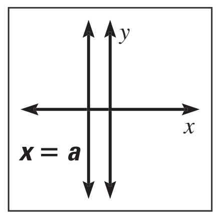
VOCABULARY

Constant function

EQUATIONS OF HORIZONTAL AND VERTICAL LINES



In the coordinate plane, the graph of $y = b$ is a ___________ line.



In the coordinate plane, the graph of $x = a$ is a ___________ line.

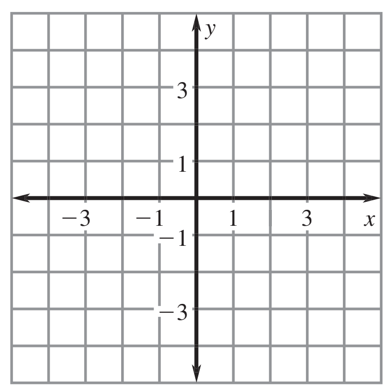
Example 1 ***Graph the Equation $y = b$***

Graph the equation $y = -3$.

The equation does not have x as a variable. The y-coordinate is always ____, regardless of the value of x. Some points that are solutions of the equation are:

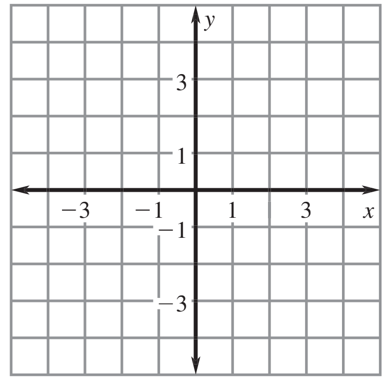
$(-3,$ ____$)$, $(0,$ ____$)$, and $(3,$ ____$)$

The graph of $y = -3$ is a _____________ line __ units ________ the _______.



Example 2 *Graph the Equation $x = a$*

Graph the equation $x = 2$.



Solution

The equation does not have y as a variable. The x-coordinate is always __, regardless of the value of y. Some points that are solutions of the equation are:

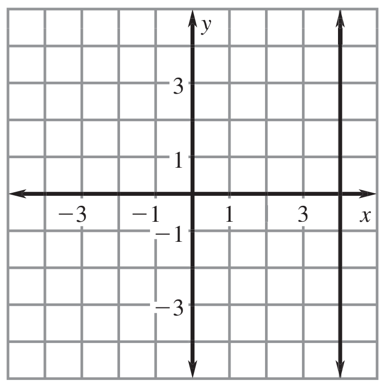
(__, −3), (__, 0), and (__, 3)

Answer The graph of $x = 2$ is a ________ line __ units to the ______ of the _______.

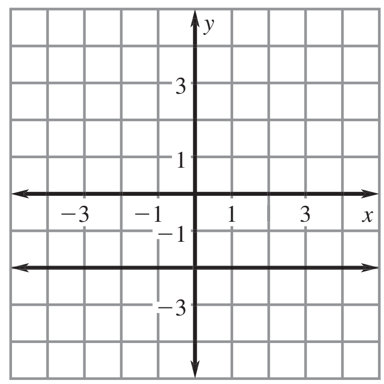
Example 3 *Write an Equation of a Line*

Write the equation of the line in the graph.

a.



b.

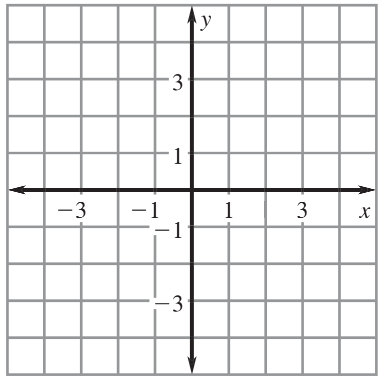
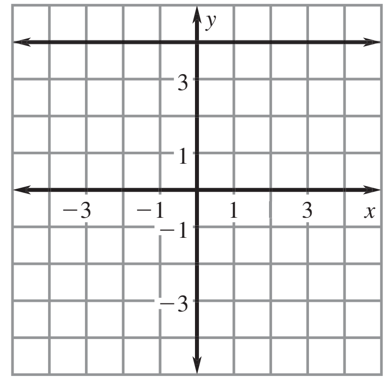


Solution

a. The graph is a ________ line. The x-coordinate is always ___. The equation of the line is _______.

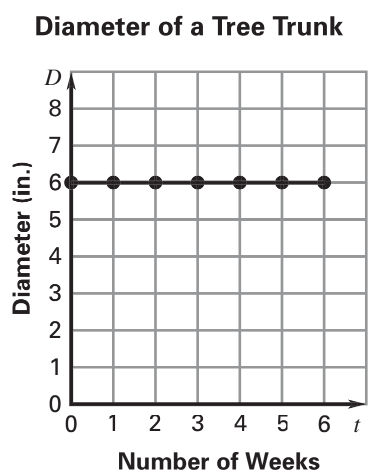
b. The graph is a ___________ line. The y-coordinate is always ____. The equation of the line is ________.

Checkpoint Complete the following exercises.

1. Graph the equation $x = -\frac{3}{2}$. 	**2.** Write the equation of the line in the graph. 

Example 4 *Write a Constant Function*

Tree Trunks The graph shows the diameter of a tree trunk over a 6-week period. Write an equation to represent the diameter of the tree trunk for this period. What is the domain of the function? What is the range?



Solution

From the graph, you can see that the diameter was about ___ inches throughout the 6-week period. Therefore, the diameter D during this time t is $D =$ ___. The domain is __________. The range is _____ ___________________.

4.4 Graphing Lines Using Intercepts

Goal Find the intercepts of the graph of a linear equation and then use them to make a quick graph of the equation.

VOCABULARY

x-intercept

y-intercept

Example 1 ***Find x- and y-Intercepts***

Find the *x*- and *y*-intercepts of the graph of the equation $-3x + 4y = 12$.

Solution

To find the *x*-intercept, substitute ___ for *y* and solve for *x*.

$-3x + 4y = 12$	**Write original equation.**
$-3x + 4(\_\_) = 12$	**Substitute ___ for *y*.**
$\_\_\_\_ = 12$	**Simplify.**
$x = \_\_\_$	**Solve for *x*.**

To find the *y*-intercept, substitute ___ for *x* and solve for *y*.

$-3x + 4y = 12$	**Write original equation.**
$-3(\_\_) + 4y = 12$	**Substitute ___ for *x*.**
$\_\_\_ = 12$	**Simplify.**
$y = \_\_$	**Solve for *y*.**

Answer The *x*-intercept is ___. The *y*-intercept is ___.

✔ ***Checkpoint*** **Complete the following exercise.**

1. Find the x-intercept and the y-intercept of the graph of the equation $2x - 5y = 10$.

The Quick Graph process works because only two points are needed to determine a line.

MAKING A QUICK GRAPH

Step 1 Find the ___________.

Step 2 Draw a coordinate plane that includes the ___________.

Step 3 Plot the ___________ and draw a line through them.

Example 2 ***Make a Quick Graph***

Graph the equation $9x + 6y = 18$.

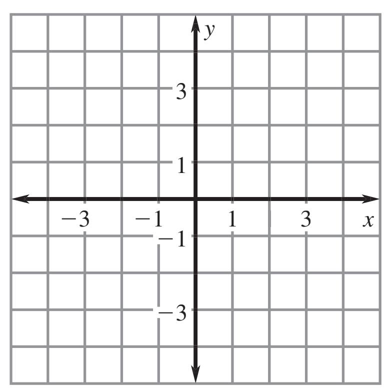
Solution

1. Find the intercepts.

$9x + 6y = 18$	**Write original equation.**
$9x + 6(\_\_) = 18$	**Substitute** ___ **for** y**.**
$x = \_\_$	**The** x**-intercept is** ___**.**
$9x + 6y = 18$	**Write original equation.**
$9(\_\_) + 6y = 18$	**Substitute** ___ **for** x**.**
$y = \_\_$	**The** y**-intercept is** ___**.**

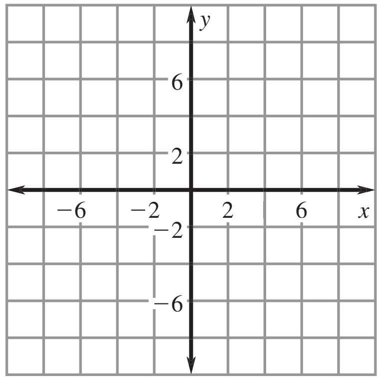
2. Draw a coordinate plane that includes the points (___, ___) and (___, ___).

3. Plot the points (___, ___) and (___, ___) and draw a line through them.



Checkpoint Complete the following exercise.

2. Graph the equation $-4x + 5y = 20$.



Example 3 ***Choose Appropriate Scales***

When you make a quick graph, find the intercepts *before* you draw the coordinate plane. This will help you find an appropriate scale on each axis.

Graph the equation $y = 5x + 35$.

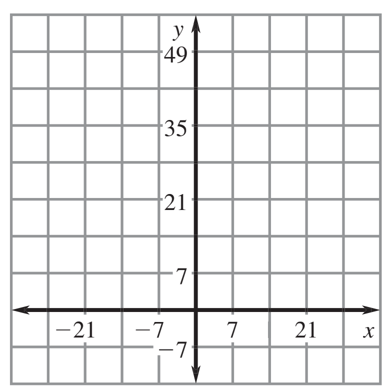
Solution

1. Find the intercepts.

$y = 5x + 35$	**Write original equation.**
$\_\_ = 5x + 35$	**Substitute __ for y.**
$\_\_\_\_\_ = 5x$	**Subtract ____ from each side.**
$\_\_\_\_ = x$	**Divide each side by ___. The x-intercept is _____ .**
$y = 5x + 35$	**Write original equation.**
$y = 5(\_\_) + 35$	**Substitute __ for x.**
$y = \_\_\_\_$	**Simplify. The y-intercept is ____ .**

2. Draw a coordinate plane that includes the points (_____, ___) and (___, ____). With these values, it is reasonable to use tick marks at _______ intervals .

3. Plot the points (_____, ___) and (___, ____) and draw a line through them.



4.5 The Slope of a Line

Goal Find the slope of a line.

VOCABULARY

Slope

Example 1 ***The Slope Ratio***

Find the slope of a ramp that has a vertical rise of 3 feet and a horizontal run of 18 feet. Let m represent the slope.

Solution

$$m = \frac{\square}{\square} = \frac{\square}{\square} = \_\_\_$$

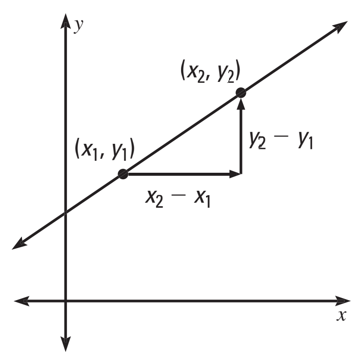
Answer The slope of the ramp is ___.

THE SLOPE OF A LINE

The slope m of a line that passes through the points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } \square}{\text{change in } \square}$$

$$= \frac{\square}{\square}.$$



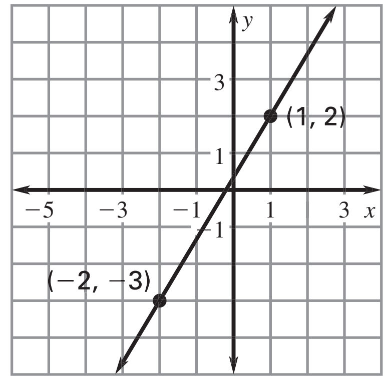
In the slope formula, x_1 is read as "x sub one" and y_2 as "y sub two."

Example 2 ***Positive Slope***

Find the slope of the line that passes through the points (1, 2) and (−2, −3).

Solution Let $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (-2, -3)$.

$m = \dfrac{\square - \square}{\square - \square}$ **Formula for slope**



$= \dfrac{\square - \square}{\square - \square}$ **Substitute values.**

$= \dfrac{\quad}{\_\_\_\_}$ **Simplify.**

$= \dfrac{\quad}{\_\_\_}$ **Slope is ________.**

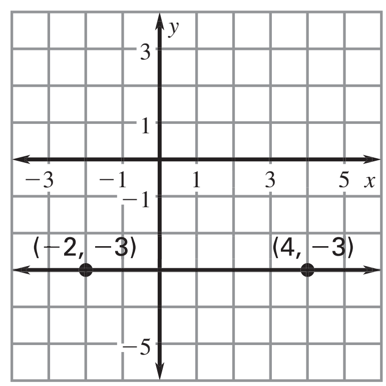
Answer The slope of the line is $\dfrac{\quad}{\_\_\_}$. The line ______ from left to right. The slope is _________.

Example 3 ***Zero Slope***

Find the slope of the line passing through the points (−2, −3) and (4, −3).

Solution Let $(x_1, y_1) = (-2, -3)$ and $(x_2, y_2) = (4, -3)$.

$m = \dfrac{\square - \square}{\square - \square}$ **Formula for slope**



$= \dfrac{\square - (\square)}{\square - (\square)}$ **Substitute values.**

$= \dfrac{\quad}{\_\_\_}$ **Simplify.**

$= \_\_\_$ **Slope is _____ .**

Answer The slope of the line is ___. The line is ___________.

Example 4 *Undefined Slope*

Find the slope of the line passing through the points (−1, −4) and (−1, −2).

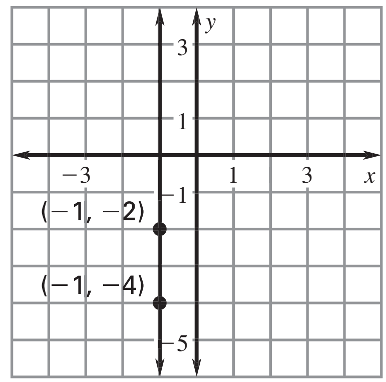
Solution

Let $(x_1, y_1) = (-1, -4)$ and $(x_2, y_2) = (-1, -2)$.

$$m = \frac{\square - \square}{\square - \square}$$ **Formula for slope**

$$= \frac{\square - (\square)}{\square - (\square)}$$ **Substitute values.**

$$= \frac{\square}{\square}$$ **Division by __ is __________.**



Answer Because division by _____ is __________, the slope is __________. The line is ________.

✔ ***Checkpoint*** **Find the slope of the line passing through the points. Then state whether the slope of the line is *positive*, *negative*, *zero*, or *undefined*.**

1. (−5, 2), (7, −2)	**2.** (0, 0), (−9, 0)
3. (−7, −8), (−7, 8)	**4.** (2, −4), (8, 6)

4.6 Direct Variation

Goal Write and graph equations that represent direct variation.

VOCABULARY

Direct variation

Constant of variation

Example 1 ***Write a Direct Variation Model***

The model for direct variation $y = kx$ is read as "y varies directly with x."

The variables x and y vary directly. One pair of values is $x = 7$ and $y = 21$.

a. Write an equation that relates x and y.

b. Find the value of y when $x = 4$.

Solution

a. Because x and y vary ________, the equation is of the form ________.

$y = kx$	**Write model for direct variation.**
____ $= k($___$)$	**Substitute** ___ **for** x **and** ____ **for** y.
___ $= k$	**Divide each side by** ___.

Answer An equation that relates x and y is ________.

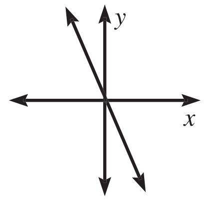
b.

$y = 3($___$)$	**Substitute** ___ **for** x.
$y =$ ____	**Simplify.**

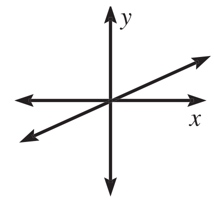
Answer When $x = 4$, $y =$ ____.

PROPERTIES OF GRAPHS OF DIRECT VARIATION MODELS

- The graph of $y = kx$ is a line through the _______ .
- The slope of the graph of $y = kx$ is ___ .



***k* is negative.**



***k* is positive.**

Example 2 ***Graph a Direct Variation Model***

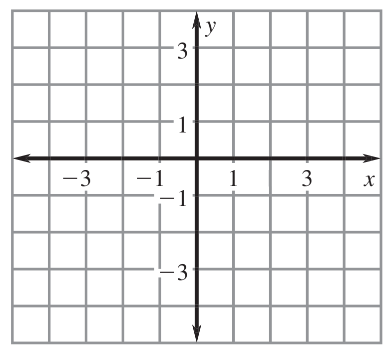
Graph the equation $y = -x$.

1. **Plot** a point at the _______.
2. **Find** a second point by choosing any value of x and substituting it into the equation to find the corresponding y-value. Use the value -3 for x.

$y = -x$	**Write original equation.**
$y = -($____$)$	**Substitute ____ for *x*.**
$y =$ ___	**Simplify.**

The second point is (____, ___).

3. **Plot** the second point and draw a line through the _______ and the second point.



✔ ***Checkpoint*** **The variables *x* and *y* vary directly. Use the given values to write an equation that relates *x* and *y*.**

1. $x = 6, y = 30$	**2.** $x = 8, y = -20$	**3.** $x = 3.6, y = 1.8$

4.7 Graphing Lines Using Slope-Intercept Form

Goal Graph a linear equation in slope-intercept form.

VOCABULARY

Slope-intercept form

Parallel lines

SLOPE-INTERCEPT FORM OF THE EQUATION OF A LINE

The linear equation $y = mx + b$ is written in **slope-intercept form,** where ___ is the slope and ___ is the y-intercept.

slope ↓ y-intercept ↓

$y =$ ___ $x +$ ___

Example 1 *Find the Slope and y-Intercept*

Find the slope and y-intercept of $-3x - y = 2$.

Solution Rewrite the equation in slope-intercept form.

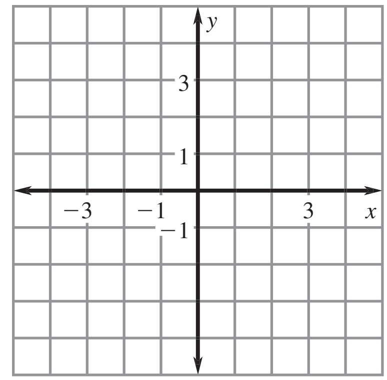
$-3x - y = 2$	**Write original equation.**
$-y =$ ____ $+ 2$	**Add ____ to each side.**
___ $=$ _________	**Divide each side by ____.**
	$m =$ ____ and $b =$ ____.

Answer The slope is ____. The y-intercept is ____.

Example 2 *Graph an Equation in Slope-Intercept Form*

Graph the equation $y = 2x - 3$.

1. **Find** the slope, ___, and the y-intercept, ____.
2. **Plot** the point $(0, b)$ when b is ____.
3. **Use** the slope to locate a second point on the line.



$$m = \frac{\square}{\square} = \frac{\text{rise}}{\text{run}} \rightarrow$$

$$\frac{\text{move } \square \text{ units up}}{\text{move } \square \text{ unit right}}$$

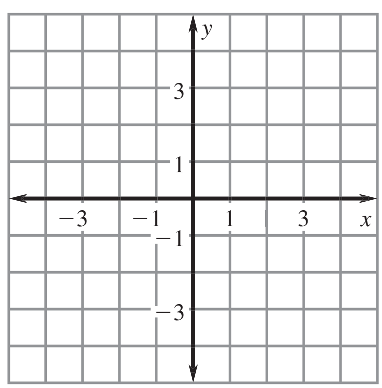
4. **Draw** a line through the two points.

✔ ***Checkpoint*** **Find the slope and y-intercept of the equation.**

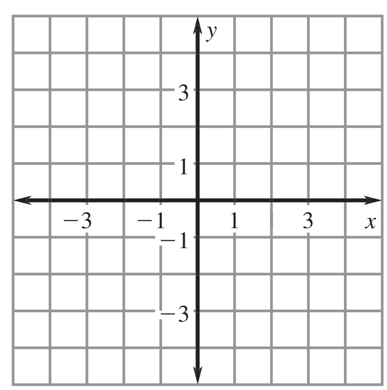
1. $y = 4 - 3x$	**2.** $2x + y = -3$	**3.** $4y = 3x - 8$

Graph the equation in slope-intercept form.

4. $y = x - 2$



5. $y = -\frac{1}{4}x + 1$



Example 3 *Identify Parallel Lines*

Which of the following lines are parallel?

line *a*: $-2x + y = 1$ **line *b*:** $2x + y = -1$ **line *c*:** $2x - y = 3$

Solution

1. **Rewrite** each equation in slope-intercept form.

   line *a*: $y =$ ________ **line *b*:** $y =$ __________ **line *c*:** $y =$ ________

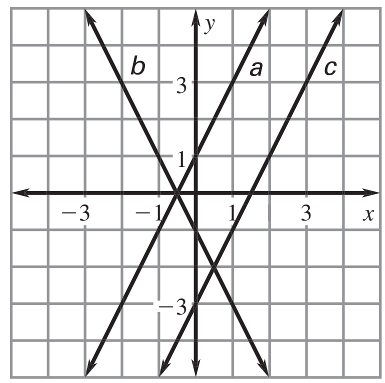
2. **Identify** the slope of each equation.

   The slope of line *a* is __. The slope of line *b* is ____. The slope of line *c* is __ .

3. **Compare** the slopes.

   Lines __ and __ are parallel because each has a slope of __.

   Line __ is *not* parallel to either of the other two lines because it has a slope of ____.



Check The graph gives you a visual check. It shows that line *b* ___________ each of the two parallel lines.

Answer Lines __ and __ are parallel.

Checkpoint **Which of the following lines are parallel?**

6. **line *a*:** $4x - 3y = 6$
   line *b*: $-8x + 6y = 18$
   line *c*: $4x + 3y = 8$

4.8 Functions and Relations

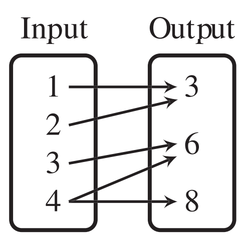
Goal Decide whether a relation is a function and use function notation.

VOCABULARY
Relation
Function
Function notation
Linear function

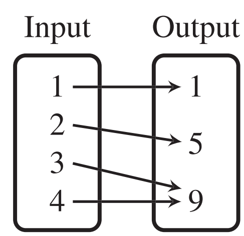
Example 1 ***Identify Functions***

Decide whether the relation is a function. If it is a function, give the domain and the range.

a.



b.



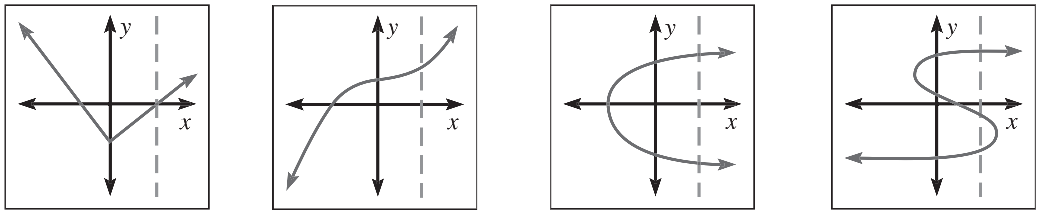
Solution

a. The relation _______ a function because _________________________ __________________.

b. The relation _______ a function. For each _______ there is _____________________. The domain of the function is _______________. The range is ____________.

VERTICAL LINE TEST FOR FUNCTIONS

A graph is a function if no ________ line intersects the graph at ______________ point.



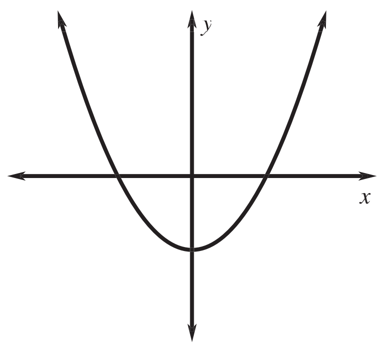
____________ ____________ ____________ ____________

____________ ____________ ____________ ____________

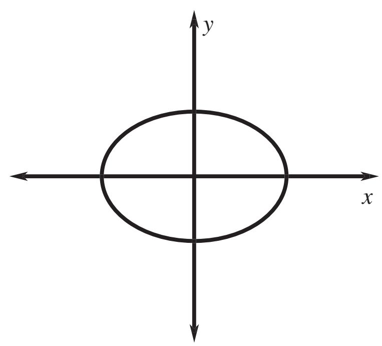
Example 2 ***Use the Vertical Line Test***

Use the vertical line test to determine whether the graph represents a function.

a.



b.



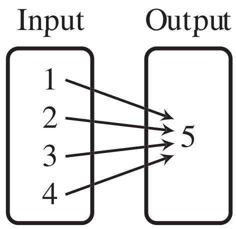
Solution

a. ____ vertical line can be drawn to intersect the graph more than once. The graph ____________________________.

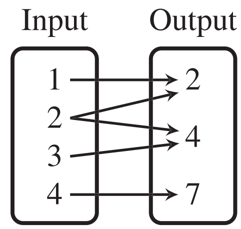
b. ____ vertical line can be drawn to intersect the graph more than once. The graph ______________________________________.

✔ ***Checkpoint*** **Decide whether the relation is a function. If it is a function, give the domain and the range.**

1.

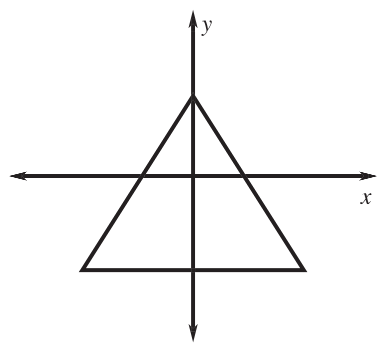


2.

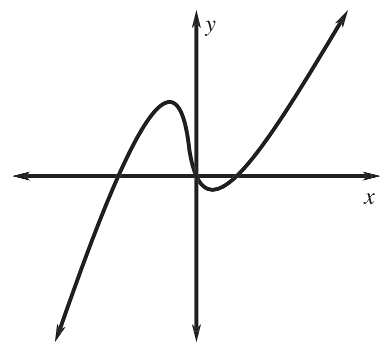


Use the vertical line test to determine whether the graph represents a function.

3.



4.



Example 3 ***Evaluate a Function***

Evaluate $g(x) = -2x + 3$ when $x = 4$.

You don't have to use f to name a function. Just as you can use any letter as a variable, you can use any letter to name a function.

Solution

$g(x) = -2x + 3$	**Write original function.**
$g(\_\_\_) = -2(\_\_\_) + 3$	**Substitute ___ for x.**
$= \_\_\_\_$	**Simplify.**

Answer When $x = 4$, $g(x) =$ ____.

Example 4 *Graph a Linear Function*

Graph $f(x) = \frac{3}{4}x - 2$.

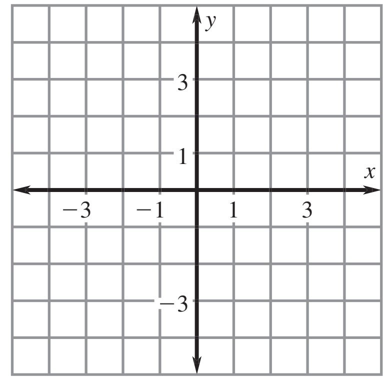
1. Rewrite the function as $y =$ __________ .

2. Find the slope and y-intercept.

$m =$ ____ and $b =$ ____

3. Use the ________ to locate a second point.

4. Draw a line through the two points.

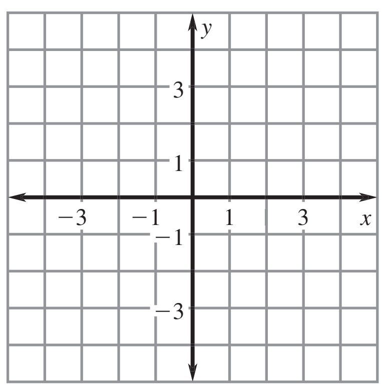


✔ *Checkpoint* Evaluate the function for the given value of the variable.

5. $f(x) = -7x + 3$ when $x = -3$	**6.** $f(x) = x^2 - 5$ when $x = 2$

Graph the linear function.

7. $f(x) = -2x + 1$



8. $f(x) = 4x - 3$